Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

MEMORANDUM

Date:	27 March 1997	TM97-02
To:	Distribution	
From:	R.D. Reasenberg	
Subject:	Some Thoughts on the Analysis of the Data from a FAME Mission.	

I. Introduction.

The following are some preliminary considerations on which to build a plan to analyze the astrometric data from FAME. This description is an outgrowth of analyses of the requirements for the study by John Chandler of the characteristics of the FAME equivalent of great-circle reductions. More recently, I have looked briefly at the analysis plan for HIPPARCOS (Perryman *et al.* 1989), and have added a few references to it below. I intend to address the connection between the FAME and HIPPARCOS reduction plans at a later time. In the present analysis, I have made no attempt to optimize the parameters of the approach described. For these parameters, I have given what I believe to be plausible values.

Other plans, possibly quite different in approach, will likely be proposed. I believe that all of the plans that merit careful consideration will use suboptimal estimation, since a direct inversion yielding five astrometric parameters for each of 10^7 stars (plus instrument parameters) is not likely to be (computationally) possible in the applicable time frame. An immediate consequence of the use of suboptimal estimation techniques is that the (complete statistical) standard deviation will not be readily available as it would be from a grand solution from a weighted least squares (WLS) estimator. The standard deviation will need to be estimated by independent means. Further, it will not be possible to rigorously evaluate the merits the proposed estimators.[1] Therefore, careful consideration should be given to the approach of the HIPPARCOS community, i.e., parallel efforts at arms length with cross checks.

The "raw data" of a FAME mission are photon counts from the CCD detectors of the spaceborne instrument. For each object observation, these raw data are reduced to a "target event" characterized by (1) the identity of the detecting CCD, (2) the estimated time at which the centroid of the target pattern exited the CCD, and (3) the target's estimated transverse position within the CCD. Such target events are the starting point for the present analysis; the centroiding

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[1] The full evaluation of a suboptimal estimator includes an effort equivalent to the development and operation of the optimal estimator. This is possible (and in many cases essential) when the suboptimal estimator has been designed to run repeatedly on a processor of limited capacity. Then it can be evaluated in a limited number of cases on a higher power machine. In fact, such evaluations may be part of the design process. In the present case, the estimator is to be used once (on the 10^7 brightest stars).

issues are not addressed here. In the discussion below, I assume the following parameters:

Field swept by the instrument (diameter of field of view)	0.75 deg
Time required to invert a 1000 parameter normal equations matrix (Using a workstation one order faster than the Sparc IPC on Chandler's desk. Such a workstation is available now, and a workstation that is an additional one order faster would likely be available at the time of the analysis.)	1 hour
Spacecraft rotation period	2 hours
Time required for the spacecraft spin to precess around the Sun direction	60 days
Stellar magnitude (used to estimate number of available stars)	photographic

In the next three sections, I describe an analysis having three stages and broadly similar to the HIPPARCOS analysis:

(Stage A) The "observing-spiral" reduction (HIPPARCOS: great-circle reduction) would address the target events collected during an interval of from two hours to a few days and yield a rotation model for the instrument during that time.

(Stage B) A global fit (HIPPARCOS: sphere solution) would interconnect the observing-spiral rotation models to yield a "global net" over the celestial sphere.

(Stage C) The application of the models and parameters determined during the first two stages (HIPPARCOS: astrometric parameter determination) to the determination of the astrometric parameters of the program stars would create the catalog.

Note that during Stages A and B, the objective is to develop a rotation model for the instrument, not to estimate the astrometric parameters of the target stars. It is only in Stage C that stellar astrometric parameters are estimated. Stage A analyses could be performed as soon as the target events are available from the first observing spiral. (Even a partial set could be used as part of the instrument check out.) Stage B analysis could first be performed well (although probably for position only) after five to eight months of data-taking, about the time that full sky coverage is first available. After about a year, the analysis could be extended to include proper motion and parallax. The merit of such analyses as a function of the data-taking span is subject to a covariance analysis that would require only modest computational effort.[2]

At all stages, the analysis must include the identification and modeling of non-point targets. (This modeling of non-point targets is not addressed here.) I believe that there will need to be iterations involving two or more stages, and an applicable scheme is described in Section V. Especially if such iteration is needed, a premium should be placed on keeping the computational load manageable. The question of global orientation and rotation is discussed in Section VI. Some discussion of the rigidity of the Stage A analysis is offered in Section VII.

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[2] I believe that a preliminary Stage B analysis could be performed a month or two after the start of the data-taking portion of the mission.

II. Observing-Spiral Reduction (Stage A).

Independent of the decision on the means of controlling the spacecraft precession, I assume here that the first stage of astrometric data reduction will incorporate the target events from a "batch interval" spanning about one day. Thus, the target events from about a dozen rotations of the instrument will be reduced together. During this interval, the center of each field of view would move along an observing spiral as the spacecraft rotates and precesses. The simultaneous reduction of data from multiple rotations of the FAME spacecraft had been considered before I began to work on the project, especially as a means to splice together spans separated by rotation-correction events. (Germain 1995, p12)

Here we make some simplifying assumptions. First, we will ignore stars brighter than mag 6 (in this analysis, but not in the real data analysis). Second, we can neglect the effect of read noise on the information from a star since we are not considering stars near the limiting magnitude. Thus the information rate of a star is proportional to its apparent brightness. Third, the stars in each one-magnitude range collectively deliver about the same amount of light. From mag 5 to mag 11 the total amount of light per one-magnitude range increases by about 40%; above mag 14, it decreases slowly. (See Allen 1976, p243.) Thus, the astrometric information rate is about the same in each one-magnitude range.

In a single rotation, the instrument's fields of view each cover about the same ≈270 sq. deg. of the celestial sphere. (Ca. 90% overlap.) During the next rotation, each covers an additional ≈80 sq. deg. After one day, the instrument has covered ≈1150 sq. deg. or 2.8% of the celestial sphere. If we include in the analysis all stars from mag 6 to mag 9, there will be about 1700 "spiral-tie stars" from one day's observing. Including the effect of multiple observations of these stars, there would be 10^4 target events.

The data would be combined in a WLS estimator. The parameter list would include two (position) parameters per spiral-tie star plus a few (or perhaps many) tens of additional parameters, mostly describing the spacecraft rotation and the instrument, including the "basic angle." Of those additional parameters, one would be an initial instrument rotation angle for the observing spiral. That angle could not be determined without some *a priori* knowledge of the positions of the observed stars, and would not be well determined until a later stage of the analysis. In fact, such *a priori* knowledge would be included in the estimator and would be required for using the cross-scan measurements of star positions to determine parameters representing the two components of the orientation of the instrument orthogonal to the spin. No attempt would be made at this stage to estimate proper motion or parallax, although *a priori* estimates of these would be used, where available, to calculate the theoretical value of the observable.

The above parameter set implies that it would take about two days to invert the full set of normal equations by ordinary means. This is neither acceptable nor necessary, as discussed below. The desired product of this first stage of the analysis is the rotation model of the instrument, not the astrometric parameters of the stars. The coefficient matrix (of the normal equations) would be particularly sparse. If the parameter set is ordered {star parameters, rotation and other parameters}, then the upper diagonal block is empty except for 2×2 blocks along the diagonal. This type of matrix invites the use of the method of "partial prereduction" (PPR, Reasenberg 1975).[3] The positions of the spiral-tie stars would be "reduced out," leaving a set of normal equations for the rotation and other parameters. When inverted, this smaller set of normal equations would yield the same parameter estimates and covariance matrix as would have been obtained from the inversion of the full set (except, of course, that the reduced result does not provide direct information for the reduced parameters). This is all that is required at this stage; the astrometric parameters of the spiral-tie stars are not needed. Use of partial prereduction would permit a larger number of spiral-tie stars.

In fact, the full set of normal equations would not be formed. A smaller set of normal equations would be set up with two parameters for a representative star and the full set of rotation and other parameters. The ($\approx 10^4$) target events would be sorted by star number. All data for the first star would be included in the normal equations along with an *a priori* position and inverse covariance for that star.[4] The PPR algorithm would be applied to remove the star parameters, making room for the next star to be included. This cycle would be repeated until all stars had been included and removed. In the end, we would have a set of normal equations for the rotation and other parameters. These would be solved in less than an hour.

We next consider the effect of increasing the faint limiting magnitude for Stage A from 9 to 10. Since the astrometric information rate is about the same in each one-magnitude range, the information would increase by 1/3 and, provided the extra stars are not used to help break degeneracy, uncertainties would decrease by a factor of $\approx$0.87. The number of stars would increase by a factor of 2.7, and the processing time would increase by about this factor. The above analysis assumes that the estimator is operating in the simple "square-root of N regime," although this has not been shown to be the case. Below some low number of stars in the analysis, there probably is degeneracy that is broken by adding extra stars. This effect is seen in the POINTS grid lock-up studies where it is characterized by the redundancy factor, M, which is the ratio of the number of observations to the number of observed stars. For the nominal case of M = 5, a typical standard deviation of stellar position decreases as 1/M, not as $1/\sqrt{M}$. (Reasenberg *et al.* 1997) The related questions of the degeneracy threshold and the rigidity of the spiral can be resolved by straightforward covariance studies once we understand the required

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[3] The partial prereduction technique appears to have been invented many times, to have been given many names, and to have been known to the HIPPARCOS investigators when they were designing their data analysis plans. It is used, for example, to eliminate uninteresting spacecraft orbital elements when spacecraft tracking data are used to estimate the coefficients of the central body gravity model. Unfortunately, I do not know a good reference to it in the open literature.

[4] Initially, the *a priori* values would come from HIPPARCOS. However, if we iterate as suggested in Section V, then we will have our own improved values to use. In the latter case, consideration will need to be given to a suitable deweighting factor.

parametric representation of the spacecraft rotation. We will then be in a better position to decide on the faint limiting magnitude for the spiral-tie stars.

III. Global Net (Stage B).

At first, this appears to be the most difficult stage in the reduction because of the large number of parameters that we might want to estimate simultaneously. For example, if we were to keep all stars to mag 8 (cf. mag 9 in Stage A), there would be 115k parameters, at five per star, plus additional parameters representing the non-point-like nature of some of the targets. For mag 7 (6), there would be 42k (15k) parameters. -- We can do better.

One approach is to consider this stage of analysis as connecting the observing spirals together using a sparse set of "global-tie stars." Here we assume that it is sufficient to have ten global-tie stars in each observing-spiral band (swath of observation) for a single rotation of the spacecraft. That's a density of 1/(27 sq. deg.), and implies that 1) a total of about 1500 global-tie stars would suffice (cf. a total of 6×10^4 spiral-tie stars -- not all considered in one analysis), and 2) each one-day batch would contain about 40 global-tie stars.[5] These stars could be selected to be bright, well distributed on the celestial sphere, and likely to be astrometrically stable. They would naturally be a subset of the spiral-tie stars. (The present analysis makes no provision for either spiral- or global-tie stars being shown to be unsuitable after selection. To first order, the mission results do not depend on the selection of the tie stars. Thus, replacement of a modest number of them later in the mission should not pose a problem.)

There would be a single (possibly iterated) fit with 7500 star parameters and 900 observing spiral initial angles. The full coefficient matrix (the matrix part of the normal equations) would require 280 MB of memory (triangular part of matrix, double precision), and its inversion would require four weeks on the nominal workstation assumed in Section I. In the case of iteration, there would be no need to form and invert the coefficient matrix after the first time. Rather, we would recompute the prefit residual with the parameters from the first iteration, form the vector part of the normal equations, and multiply it by the saved inverse coefficient matrix. -- We can do even better.

As done in the previous section, for the present WLS fit we would apply the PPR method as the normal equations are formed from the ($\approx 10^5$) target events of the global-tie stars, which would be sorted by star number. The (5) astrometric parameters of each global-tie star would be reduced out, leaving a set of normal equations for both the observing-spiral initial angles and a (presumably small) set of instrument parameters. When inverted, these normal equations would yield the same parameter estimates and covariance matrix as would have been obtained from the inversion of the full set, as previously noted. This is all that is required at this stage. Use of PPR

[5] It might be more logical to specify the number of global-tie stars per batch of data, i.e., per full (one day) observing spiral band than their density. The required number and how to specify it will require study.

would permit both a larger number of stars and a larger number of initial angles than suggested above. The latter would be particularly valuable if we find that the batch interval needs to be considerably shorter than one day. (We will not be able to address the batch interval until we better understand the rotation dynamics of the spacecraft. In fact, for a robust mission plan, we must be prepared to perform the data analysis even if we find after launch that the batch intervals need to be shorter than planned.)

An extension of the basic PPR technique (Chandler 1989) permits the determination of the reduced parameters and the corresponding covariance. Using the extended technique amounts to applying a specialized knowledge of the structure of a sparse matrix to speed up its inversion. However, there is no need to use the extended technique here or to determine the astrometric parameters of the global-tie stars in support of the central goal, the determination of the parameters of the program stars. The extended technique would, however, provide a means of investigating the uncertainties (five parameter covariance) in the astrometric star parameters for a representative set of global-tie stars.

IV. The Rest of the Stars (Stage C).

Typically, a star will be observed several tens of times during a mission. With the global net developed, the astrometric parameters of the stars can be estimated easily by the following suboptimal (but perhaps good enough) method with three steps. 1) The 10^9 target events are sorted by star. 2) The event times of a star are combined with the saved information about the instrument rotation to yield precise single-direction star positions. 3) For each star in turn, a WLS fit yields the five astrometric parameters (or a larger number of parameters if the "star" had structure, e.g., was part of a multi-star system.)

V. A Possible Near-Global Iteration.

Nearly all of the tie stars would be mag 9 or brighter, and thus well measured by HIPPARCOS. Thus, we would have reasonably good *a priori* values for their astrometric parameters, for example, from HIPPARCOS.

To iteratively clean up the solution, the Stage-C procedure could be applied to the combined set of spiral- and global-tie stars only. With improved estimates of positions, proper motions and parallaxes for these tie stars, the observing-spiral reductions and global fit could be repeated. Unless this iteration uncovers and precipitates the removal of bad data (blunders), it is hard for me to see (at this early stage of the analysis) how it would need to be repeated. However, we need to investigate the convergence rate of this scheme.

As a practical matter, we would likely want to perform such an iteration several times during the mission to improve the catalog that forms the basis for various near-real-time analyses. The first such iteration could be after about six (or eight) months of data taking, when we first have full sky coverage. (It may be possible to perform a preliminary iteration much earlier.) These additional analyses would serve to detect problems early in the mission.

VI. Connecting to the Nominal Reference Frame.

For many astrophysical purposes, an arbitrary orientation shift of the coordinate frame is of no concern. However, particularly for the Navy applications of the FAME catalog, a "correct" orientation would be valuable. One way of achieving this is to use a pre-existing catalog to provide *a priori* estimates of the star positions.[6] These could be mild enough that they would not significantly bias the inter-star distance estimates, but strong enough to break the rotational near degeneracy of the normal equations. (The degeneracy would be broken automatically by the aberration terms, which would connect the stellar frame to the Earth's orbit.)

As a variant of this scheme, the frame orientation and rotation could be tied to a few distant objects, most advantageously to quasars. This could be done by including a set of quasars along with the global-tie stars in the fit of the global net. The quasars would be given *a priori* positions and (presumably zero) proper motions in accord with the best determinations available. This approach has the advantage of connecting the FAME frame to the best candidate inertial references.

VII. Observing-Spiral Reduction, Discussion and a Variation.

In the scheme described in Sections II and III, data taken during a batch interval of about one day is processed to yield the rotation model of the instrument modulo an initial angle. That initial angle cannot be determined from the measurements made during the batch interval, except by using *a priori* knowledge of the star positions. The initial angle for each observing spiral is found during the global net estimation. Implicit in this description is the assumption that the observing-spiral reduction yields a "rigid" unit, e.g., that there is not an accumulating error from one spacecraft rotation to the next. This issue will be addressed in Chandler's study.

The section of the observing-spiral band (swath of observation) for a single rotation of the spacecraft is shifted from the band of the previous (or following) spacecraft rotation by the precession of the spin axis. The overlap of the bands ranges from 100% where they cross to about 50%, assuming smooth precession that does not require frequent spin-correction events. The overlap from the first to last band is smaller, about 17 sq. deg. However, the expected number of stars (mag 6 to 9) in this region is 25. Thus, in selecting the spiral-tie stars, extra stars could be selected in these overlap regions should covariance studies confirm that they are critical, as I suspect. These extra stars would provide an enhanced direct connection to supplement the indirect connections that use stars in intermediate band sections. This description suggests that a

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[6] This may be the only viable approach. The standard reference frames are connected to the rotation and orbit of Earth. A spaceborne astrometric mission is insensitive to the former and only mildly sensitive to the latter.

observing-spiral reduction can be made to yield a rigid spiral that can reasonably be represented by the single observing spiral initial angle.

The batch interval need not be one day. Covariance studies need to be performed to determine the pros and cons of different lengths. Further, it may be advantageous for the batches to overlap, provided the data in the overlap are not double counted. For example, with an overlap of four rotation periods, the data taken during the last and first two spacecraft rotations would be discarded after the observing-spiral reduction, as would be the part of the rotation model applicable to those data. During the observing-spiral reduction, those data would help tie together the data near the ends of the batch interval. This would be particularly useful if the rotation-correction events are frequent.

VIII. References.

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IX. Acknowledgments

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X. Distribution

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